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Surprises in Open-String Perturbation Theory

Augusto Sagnotti

*Dipartimento di Fisica
Università di Roma "Tor Vergata"
I.N.F.N. - Sezione di Roma "Tor Vergata"
Via della Ricerca Scientifica, 1 00133 Roma ITALY*

Abstract

The perturbative analysis of models of open and closed superstrings presents a number of surprises. For instance, variable numbers of antisymmetric tensors ensure their consistency via generalized Green-Schwarz cancellations and a novel type of singularity occurs in their moduli spaces. All these features are related, in one way or another, to the presence of boundaries on the world sheet or, equivalently, of extended objects (branes) interacting with the bulk theory in space time. String dualities have largely widened the interest in these models, that exhibit a wealth of generic non-perturbative features of String Theory.

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Surprises in Open-String Perturbation Theory

Augusto Sagnotti^{a*}

^aDipartimento di Fisica, Università di Roma “Tor Vergata”
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1. INTRODUCTION

The basic rules determining the perturbative spectra of oriented closed strings essentially amount to two basic constraints on their torus amplitudes, *modular invariance* and *spin-statistics* [1]. The former makes the choice of world-sheet “time” immaterial, while the latter fixes the relative contributions of Bose and Fermi fields. By now familiar for almost a decade, these rules determine the *GSO* projections [2] for vacua in several space-time dimensions with different numbers of space-time supersymmetries, and have deepened considerably our understanding of String Theory. On the other hand, models of open and closed strings, the very ones that marked the birth of the field [3], have long been of interest only to a small community [4]. To some extent, this uneven fate is to be traced to the restrictions on their gauge groups [5] and to the relative simplicity and elegance of the heterotic string [6].

In 1987 I suggested that a link between open-string models and *symmetric* models of oriented closed strings, exhibited by the only two examples known at the time, the bosonic string and the $SO(32)$ type-I model [7], be taken as a general building principle for their spectra [8]. The resulting connection, a parameter-space analogue of

the orbifold construction [9], relates the type-IIB model of oriented closed strings [10] to its “open descendant”, the $SO(32)$ model of Green and Schwarz. Subsequent work exhibited the role of boundary conditions in lower-dimensional models [11] and led to new classes of open-string spectra in ten and six dimensions [12], with corresponding patterns of Chan-Paton symmetry breaking.

Six-dimensional models promptly displayed a rewarding surprise: their low-energy spectra include variable numbers of antisymmetric tensors [12] that take part in a generalized anomaly cancellation mechanism [13]. These marked differences with respect to perturbative vacua of the heterotic string were correctly traced to the “parent” type-IIB string. Moreover, an analysis of the low-energy supergravity revealed the existence of peculiar singularities in the moduli spaces of tensor multiplets, that result in infinite gauge coupling constants [13]. This *perturbative* phenomenon, and its non-perturbative counterpart expected to present itself in heterotic models [14], have aroused some interest during the past year. The singularities reflect the presence in the vacuum of string excitations with vanishing tension [15], indicating some sort of string generalization of the Nambu-Jona Lasinio-Goldstone phenomenon. More general types of compactifications, with non-trivial vacuum values for scalar fields in the internal space, have been advocated to provide a geometrical framework for these re-

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sults [16].

String dualities [17], and in particular the strong-weak coupling relation in ten dimensions between the type-I $SO(32)$ model and the $Spin(32)/Z_2$ heterotic string [18], have turned open strings into a central ingredient of the emerging picture. Most notably, Polchinski's observation [19] that extended objects, D branes, where open-string endpoints terminate, have a central role as carriers of Ramond-Ramond charges, has provided a conformal field theory setting for some of the soliton studies [20] that have played a central role in the development of the string duality picture. The recent surge of interest in open-string vacua, accompanied by the emergence of a new term, "orientifold", that stresses the geometrical significance of this peculiar orbifold construction, has led to the construction of many more six-dimensional models [21–24]. Four-dimensional vacua with $N = 1$ supersymmetry are comparatively less understood [25,26], but [26] contains the first instance of a chiral model, with three generations of chiral fermions. The Kähler manifold of the untwisted scalars in the unoriented closed sector, $Sp(8)/(SU(4) \times U(1))$, is strongly suggestive of a twelve-dimensional interpretation in the spirit of [16].

The relation between closed models and their open descendants, a general property of Conformal Field Theory and current algebra, associates one or more classes of "descendants" [27,28] to the ADE series [29] of minimal and $SU(2)$ WZW models [30]. The resulting models, also of some potential relevance to Condensed Matter Physics [31], reveal the occurrence of another exotic phenomenon: the open sector may possess a *larger* symmetry than the bulk theory [28].

This article is based on seminars presented at CERN, at the XXVIII Institute d'Ete of the Ecole Normale Supérieure in Paris and at the XXX Ahrenschoop Symposium in Buckow. Since much literature is now available on D branes and their applications [32], I shall confine my attention to some of the "miracles" of open-string perturbation theory that have shaped my own understanding of the subject. In the next Section I review briefly the scope of the original proposal,

while displaying, in a couple of simple instances, the resulting structure of open-string spectra. The following Section illustrates the nature of the generalized Green-Schwarz mechanism, while displaying some corresponding properties of six-dimensional supergravity. The final Section describes, in the open descendants of the D_5 model of [29], the extension of internal symmetry in the open sector via simple currents of non-integer dimension [28].

2. OPEN DESCENDANTS, OR ORIENTIFOLDS

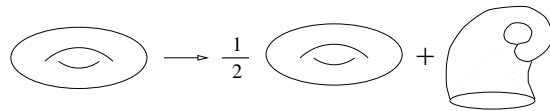


Figure 1. Projection of the closed spectrum

Let me begin by reviewing the original proposal of [8]. The basic idea (fig. 1) is to use world-sheet parity to project a consistent spectrum of oriented closed strings symmetric under the interchange of left and right modes. This operation mixes left and right modes, and its proper completion requires the Klein-bottle amplitude. As this violates modular invariance, singular (ultraviolet) contributions can result from the origin, the edge of moduli space, that in this case is a half-line. On the other hand, the Klein-bottle amplitude is "dual" to an amplitude describing the propagation of closed strings between two cross-caps, and the modular transformation relating these two pictures turns the ultraviolet divergences into infrared divergences related to massless exchanges. A suitable spectrum of open strings, the "twisted sector" of this construction, can dispose of the resulting singularities, thus restoring the consistency of the models. Indeed, Klein-bottle, annulus and Möbius strip, though vastly different, are all related by suitable modular transformations to "dual" amplitudes describing closed-string exchanges between pairs of boundaries and/or crosscaps². In particular (fig.

²In order to add a crosscap to a surface, one cuts from it a disk and glues to the resulting hole the single boundary of a Möbius strip.

2), the annulus corresponds to a tube terminating at two boundaries, while the Möbius strip corresponds to a tube terminating at one boundary and one crosscap. As a result, the proper factorization of these amplitudes turns the consistency conditions into a set of linear equations (tadpole conditions) for the numbers of “colors”, or types of Chan-Paton charges [5], associated to the various types of boundaries. Some of the tadpole conditions are related to unphysical exchanges and signal the presence of anomalies [33].

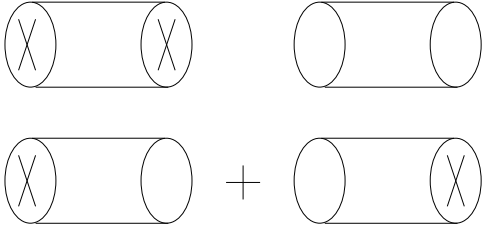


Figure 2. Klein bottle, annulus and Möbius strip

It is amusing to see how this pattern is realized, leading to whole new classes of conformal models. Let me begin with a few simple cases, and in particular with two classes of non-supersymmetric ten-dimensional models drawn from [12] and [34]. The models in the first class are described by

$$\mathcal{T} = |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2 \quad (1)$$

$$\mathcal{K} = \frac{1}{2} (O_8 + V_8 - S_8 - C_8) \quad (2)$$

$$\mathcal{A} = \frac{n_o^2 + n_v^2 + n_s^2 + n_c^2}{2} V_8 + (n_o n_v + n_s n_c) O_8 - (n_v n_s + n_o n_c) S_8 - (n_v n_c + n_o n_s) C_8 \quad (3)$$

$$\mathcal{M} = -\frac{n_o + n_v + n_s + n_c}{2} \hat{V}_8, \quad (4)$$

where the characters of level-one $SO(8)$ modules are (orthogonal) combinations of fourth powers of Jacobi theta functions and of the Dedekind η function

$$\begin{aligned} O_8 &= \frac{\theta_3^4 + \theta_4^4}{2\eta^4} & V_8 &= \frac{\theta_3^4 - \theta_4^4}{2\eta^4}, \\ S_8 &= \frac{\theta_2^4 + \theta_1^4}{2\eta^4} & C_8 &= \frac{\theta_2^4 - \theta_1^4}{2\eta^4}, \end{aligned} \quad (5)$$

and where the “hatted” functions, of argument $-\sqrt{q}$, are redefined by phases and start with posi-

tive coefficients [12,27]. In the unprojected closed sector, $|O_8|^2$ starts with a tachyon T and $|V_8|^2$ starts with the usual collection of $g_{\mu\nu}$, $B_{\mu\nu}$ and ϕ . In addition, there are two low-lying sectors of RR bosons, $A^+_{\mu\nu\rho\sigma}$, a four-form with self-dual field strength, a two-form $B'_{\mu\nu}$ and a scalar ϕ' from $|S_8|^2$, and $A^-_{\mu\nu\rho\sigma}$, a four-form with antiself-dual field strength, a two-form $B''_{\mu\nu}$ and a scalar ϕ'' from $|C_8|^2$. After the projection, one is left with g , ϕ and T from the NS-NS sectors, and B' and B'' from the $R-R$ sectors. The open sector of eqs. (3) and (4) is chiral, since the left-handed fermions in S_8 and C_8 carry different types of quantum numbers. The tadpole conditions identify the dimensionalities of pairs of Chan-Paton charge spaces, setting $n_o = n_v$ and $n_s = n_c$. The model includes *two* two-forms, both involved in the anomaly cancellation mechanism, since the residual anomaly polynomial is

$$\begin{aligned} I_A \sim & (\text{tr} F_o^2 - \text{tr} F_v^2) \left(\text{tr} F_s^4 - \text{tr} F_c^4 \right. \\ & \left. - \frac{1}{4} \text{tr} R^2 (\text{tr} F_s^2 - \text{tr} F_c^2) \right) \\ & + (\text{tr} F_s^2 - \text{tr} F_c^2) \left(\text{tr} F_o^4 - \text{tr} F_v^4 \right. \\ & \left. - \frac{1}{4} \text{tr} R^2 (\text{tr} F_o^2 - \text{tr} F_v^2) \right). \end{aligned} \quad (6)$$

Thus, B' and B'' share in a democratic fashion the crucial task of canceling the gauge anomalies, while there are no gravitational anomalies since there is no net number of chiral fermions.

A variant, corresponding to a different choice of Klein-bottle projection, was introduced in [34]. It is particularly interesting since it includes a $U(32)$ model that is not supersymmetric but is free of tachyons, an open analogue of the $SO(16) \times SO(16)$ model of [35]. Indeed, the choice of Klein-bottle projection, determined by the “crosscap constraint” [27], is *not* unique in general, and the different choices correspond to vastly different spectra. In this case, taking

$$\mathcal{K} = \frac{1}{2} (-O_8 + V_8 + S_8 - C_8), \quad (7)$$

the projected closed spectrum contains g and ϕ (but *no* tachyon) from the $NS-NS$ sectors, and A^+ , ϕ' and B'' from the $R-R$ sectors.

The corresponding chiral open sector, described by

$$\mathcal{A} = n \bar{n} V_8 - \frac{n^2 + \bar{n}^2}{2} C_8 \quad (8)$$

and

$$\mathcal{M} = \frac{n + \bar{n}}{2} \hat{C}_8 \quad , \quad (9)$$

can be obtained permuting some of the charge assignments in eq. (3) and turning pairs of “real” charge spaces into pairs of “complex” spaces for the (anti)fundamental representations of a unitary group. The corresponding multiplicities are equal in pairs, and in this case $n = \bar{n} = 32$. This recurrent possibility, related to the presence of simple currents³ in the spectrum, was first noted in [12] and is studied in detail in [27].

The residual anomaly polynomial of this model

$$\begin{aligned} I_A \sim & (\text{tr} F^2 - \text{tr} R^2) \left(\text{tr} F^4 + \frac{1}{8} \text{tr} R^4 \right. \\ & + \frac{1}{32} (\text{tr} R^2)^2 - \frac{1}{8} \text{tr} F^2 \text{tr} R^2 \Big) \\ & + \frac{2}{3} \left(\text{tr} F^3 - \frac{1}{8} \text{tr} F \text{tr} R^2 \right)^2 \\ & + \text{tr} F \left(\frac{2}{5} \text{tr} F^5 + \frac{1}{240} \text{tr} F \text{tr} R^4 \right. \\ & \left. \left. - \frac{1}{192} \text{tr} F (\text{tr} R^2)^2 \right) \right) \quad (10) \end{aligned}$$

is particularly interesting. Together with the two-form B'' , two more fields take part in a generalized Green-Schwarz mechanism, A^+ and the eight-form dual to ϕ' . The coupling of the latter gives mass to the $U(1)$ vector field, and the actual low-energy gauge group is thus $SU(32)$. It should be noticed, in particular, that the self-duality of the field strength of A^+ reflects itself in the nature of the corresponding term in eq. (10), a perfect square. We have thus come across the first miracle alluded to in the Introduction: the Green-Schwarz mechanism takes typically a more general form in these models, with contributions from forms of different degrees. This is actually the most complicated example I am aware of,

³Simple currents are operators with abelian fusion rules with all other fields in the theory. This notion was introduced in [36].

since the six-dimensional ones of [13] only include (anti)self-dual two-forms, and occasionally four-forms, in a way that extends the original four-dimensional mechanism of [37].

Let me now turn to a couple of additional instances. The first presents itself when one compactifies the type-I superstring to nine dimensions, and will serve the purpose of illustrating the role of D branes in connection with T -duality. In this case

$$\mathcal{T} = |V_8 - S_8|^2 \frac{\sum q^{\frac{\alpha'}{4} p_L^2} \bar{q}^{\frac{\alpha'}{4} p_R^2}}{\eta(q)\bar{\eta}(\bar{q})} \quad , \quad (11)$$

where, as usual,

$$p_{L,R} = \frac{m}{R} \pm \frac{nR}{\alpha'} \quad , \quad (12)$$

describes the toroidal compactification of the Type-IIB superstring to nine dimensions. In the absence of Wilson lines, the familiar Klein-bottle projection

$$\mathcal{K} = \frac{1}{2} (V_8 - S_8) \frac{\sum q^{\alpha' \frac{m^2}{2R^2}}}{\eta(q^2)} \quad (13)$$

leads to

$$\mathcal{A} = \frac{N^2}{2} (V_8 - S_8) \frac{\sum q^{\alpha' \frac{m^2}{2R^2}}}{\eta(\sqrt{q})} \quad (14)$$

and

$$\mathcal{M} = - \frac{N}{2} (\hat{V}_8 - \hat{S}_8) \frac{\sum q^{\alpha' \frac{m^2}{2R^2}}}{\hat{\eta}(-\sqrt{q})} \quad , \quad (15)$$

where $N = 32$ and, again, the power series of “hatted” functions starts with unit coefficient. The spectrum is clearly *not* invariant under T duality, since \mathcal{K} , \mathcal{A} and \mathcal{M} are all drastically altered by the usual replacement of R with α'/R . This should be contrasted with the familiar case of the heterotic string, where T duality finds an elegant setting in the Narain lattice construction [38]. One can actually explain in rather simple terms what is happening [39]. Indeed, T duality is an asymmetric parity operation on the world sheet, that transforms the (anti)holomorphic portions of the internal string coordinate according to $X_L \rightarrow X_L$ and $X_R \rightarrow -X_R$. Since the

usual Neumann condition essentially associates the open-string sector to $X_L + X_R$, a T duality turns it into a "Dirichlet" open sector corresponding to $X_L - X_R$.

Where do the "Dirichlet" ends terminate? We owe to [19] a pervasive geometrical characterization of these extended objects, now commonly referred to as D branes. In this case, they are just hyperplanes, but a closer inspection reveals that they are actually dynamical objects, a variety of string solitons. There is already a vast literature on this important topic [32], and I shall thus confine my attention, as advertised, to their role in *perturbative* open-string spectra. Indeed, the first lesson is precisely that they *are* part of perturbative open-string spectra, so what better way do we have to gain some familiarity with their properties! The second observation is that, allowing for these objects, one is apparently going past the original restriction [8] to models with a left-right symmetric GSO projection. Actually, insofar as one is ready to perform suitable T -duality transformations, every situation is captured by the original setting. There is a simple, canonical example, that I find quite instructive. It is a generalization of the toroidal model of eqs. (14)-(15) where, following [40], one allows for Wilson lines breaking $SO(32)$ to $U(M) \times SO(32 - 2M)$. The Wilson lines affect the energy levels via minimal couplings, so that

$$\begin{aligned} \mathcal{A} = & \left(\frac{N^2}{2} + M\bar{M} \right) (V_8 - S_8) \frac{\sum q^{\alpha' \frac{m^2}{2R^2}}}{\eta(\sqrt{q})} \\ & + NM (V_8 - S_8) \frac{\sum q^{\alpha' \frac{(m+aR/2)^2}{2R^2}}}{\eta(\sqrt{q})} \\ & + N\bar{M} (V_8 - S_8) \frac{\sum q^{\alpha' \frac{(m-aR/2)^2}{2R^2}}}{\eta(\sqrt{q})} \\ & + \frac{M^2}{2} (V_8 - S_8) \frac{\sum q^{\alpha' \frac{(m+aR)^2}{2R^2}}}{\eta(\sqrt{q})} \\ & + \frac{\bar{M}^2}{2} (V_8 - S_8) \frac{\sum q^{\alpha' \frac{(m-aR)^2}{2R^2}}}{\eta(\sqrt{q})} , \quad (16) \end{aligned}$$

$$\mathcal{M} = -\frac{N}{2} (\hat{V}_8 - \hat{S}_8) \frac{\sum q^{\alpha' \frac{m^2}{2R^2}}}{\hat{\eta}(-\sqrt{q})}$$

$$\begin{aligned} & - \frac{M}{2} (\hat{V}_8 - \hat{S}_8) \frac{\sum q^{\alpha' \frac{(m+aR)^2}{2R^2}}}{\hat{\eta}(-\sqrt{q})} \\ & - \frac{\bar{M}}{2} (\hat{V}_8 - \hat{S}_8) \frac{\sum q^{\alpha' \frac{(m-aR)^2}{2R^2}}}{\hat{\eta}(-\sqrt{q})} . \quad (17) \end{aligned}$$

where $N + 2M = 32$.

The Möbius contribution is only associated to terms of \mathcal{A} with identical charges, but the corresponding momentum shifts are doubled, since in the proper parametrization its boundary has twice the length of each boundary of the annulus. All this is nicely consistent with the factorization of "dual" vacuum channels.

If one performs a T duality transformation, the translations in momentum space turn into translations in coordinate space. As these are simple to visualize (fig. 3), the resulting D -brane picture, though equivalent, has undoubtedly some aesthetic appeal. For instance, it is evident from eqs. (16) and (17) that $a = \frac{1}{R}$ results in an enhancement (or rather, a partial recovery) of the gauge symmetry, that becomes $SO(32-2M) \times SO(2M)$, while $a = \frac{2}{R}$ retrieves the full $SO(32)$ gauge group. Referring to fig. 3, in the T -dual picture the prototype breaking of eqs. (16) and (17) involves strings stretched between a D brane (D_o) at the bottom of the circle and an additional pair of D branes at points symmetrically located with respect to the vertical (D_+ and D_-). The partial recovery of gauge symmetry takes place when the last two meet at the top of the circle, whereas full recovery is attained when all three meet at the bottom.

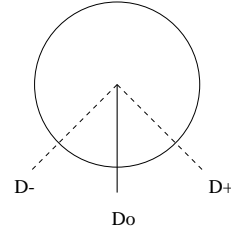


Figure 3. D-brane picture of Wilson-line breaking

One can actually modify the Klein-bottle projection, since the crosscap constraint [27] allows one to (anti)symmetrize internal momenta according to their quantum numbers. This interest-

ing possibility, first exploited in [22] and discussed further in [24], corresponds to

$$\mathcal{K} = \frac{1}{2} (V_8 - S_8) \frac{\sum (-1)^m q^{\alpha' \frac{m^2}{2R^2}}}{\eta(q^2)} \quad (18)$$

and leads to open descendants *without* an open sector. Indeed, familiar properties of theta functions imply that the transverse-channel amplitude corresponding to eq. (18) does not introduce any singularities, since it does not involve any massless exchange.

Let me now complicate matters slightly. Following [11], I shall describe how to associate open descendants to $c = 1$ orbifolds. While the resulting conformal models can be associated to string compactifications, for simplicity I shall only consider the internal part. The novelty is that strings with Neumann and Dirichlet conditions coexist in the spectrum, independently of T dualities. This geometrical setting has been discussed for the superstring in [21], with resulting constraints equivalent to those in [11]. The covariant form has the advantage of leading naturally to large Chan-Paton gauge groups. Indeed, in rational models one typically faces the presence of *quantized* backgrounds for the $NS - NS$ two-form, and these reduce the rank of the Chan-Paton group. For instance, starting from a total dimensionality 32 for the Chan-Paton charge space, a quantized two-form of (even) rank r results in effective quantized Wilson lines that reduce the dimensionality of the charge space to $32/2^{r/2}$ [40].

The oriented closed sector [9] is built projecting the internal part of the torus amplitude (11) so as to enforce the identification between X and $-X$ required by the geometry of the orbifold. The resulting “untwisted” sector breaks modular invariance, that is recovered associating a pair of “twisted” sectors to the two fixed points of the orbifold. The resulting torus amplitude is

$$\begin{aligned} \mathcal{T} = & \frac{1}{2} \frac{\sum q^{\frac{\alpha'}{4} p_L^2} \bar{q}^{\frac{\alpha'}{4} p_R^2}}{\eta \bar{\eta}} + \frac{1}{2} \left| \frac{\theta_3 \theta_4}{\eta^2} \right| \\ & + \frac{1}{2} \left| \frac{\theta_2 \theta_3}{\eta^2} \right| + \frac{1}{2} \left| \frac{\theta_2 \theta_4}{\eta^2} \right|. \end{aligned} \quad (19)$$

In order to construct the unoriented closed sector of the descendant, one starts again from the

Klein bottle amplitude. In this case, however, an inspection of the sections allowed on the genus-one surfaces [11] or, equivalently, of the operator content, reveals the need for both momentum and winding sums. As a result

$$\begin{aligned} \mathcal{K} = & \frac{1}{4} \frac{\sum_m q^{\alpha' \frac{m^2}{2R^2}}}{\eta(q^2)} + \frac{1}{4} \frac{\sum_n q^{\frac{n^2 R^2}{2\alpha'}}}{\eta(q^2)} \\ & + \left(\frac{\theta_2 \theta_3}{\eta^2} (q^2) \right)^{1/2} \end{aligned} \quad (20)$$

is invariant under T duality transformations. The unitarity of the vacuum channels exposed by an S transformation then requires that the open sector contain momentum and winding sums as well. Therefore, one has to face the simultaneous presence of NN and DD strings, since the former can have momentum, while the latter can have windings in the internal direction. It should be appreciated, however, that the endpoints of these DD strings sit at the two fixed points of the orbifold. There are thus three types of DD strings: with both ends at one or the other fixed point, or stretched between the two fixed points. This is a discrete version of the breaking of fig. 3, as pertains to the absence of corresponding moduli in the projected spectrum. Aside from the usual NN strings (possibly with “quantized” Wilson lines), there are additional ones with one N and one D end, with the latter at any of the two fixed points. In conclusion,

$$\begin{aligned} \mathcal{A} = & \frac{tr(\mathbf{1}_N)^2}{4} \frac{\sum_m q^{\alpha' \frac{m^2}{2R^2}}}{\eta} \\ & + \frac{tr(\mathbf{R}_N)^2}{4} \left(\frac{\theta_3 \theta_4}{\eta^2} \right)^{1/2} \\ & + \sum_{i=1,2} \frac{tr(\mathbf{1}_N) tr(\mathbf{1}_{Di})}{4} \left(\frac{\theta_2 \theta_3}{\eta^2} \right)^{1/2} \\ & + \sum_{i=1,2} \frac{tr(\mathbf{R}_N) tr(\mathbf{R}_{Di})}{4} \left(\frac{\theta_2 \theta_4}{\eta^2} \right)^{1/2} \\ & + \sum_{i,j=1,2} \frac{tr(\mathbf{1}_{Di})^2}{4} \frac{\sum_n q^{\alpha' \frac{(n+(i-j)/2)R^2}{2\alpha'}}}{\eta} \\ & + \sum_{i=1,2} \frac{tr(\mathbf{R}_{Di})^2}{4} \left(\frac{\theta_3 \theta_4}{\eta^2} \right)^{1/2} \end{aligned} \quad (21)$$

and

$$\begin{aligned} \mathcal{M} = & \frac{tr(\mathbf{1}_N) + \sum_{i=1,2} tr(\mathbf{1}_{Di})}{4} \left(\frac{\hat{\theta}_3 \hat{\theta}_4}{\hat{\eta}^2} \right)^{1/2} \\ & + \sum_{i=1,2} \frac{tr(\mathbf{1}_{Di})}{4} \frac{\sum_n q^{\alpha' \frac{(n+(i-j)/2)R^2}{2\alpha'}}}{\hat{\eta}} \\ & + \frac{tr(\mathbf{1}_N)}{4} \frac{\sum_m q^{\alpha' \frac{m^2}{2R^2}}}{\hat{\eta}}, \end{aligned} \quad (22)$$

where $tr(\mathbf{1}) = Q_+ + Q_-$ and $tr(\mathbf{R}) = Q_+ - Q_-$ implement a Z_2 breaking in the charge sectors. For brevity, I have omitted the arguments of all θ and η functions, equal to \sqrt{q} in the annulus amplitude and to $-\sqrt{q}$ in the Möbius amplitude.

It can be verified that the vacuum channels of \mathcal{K} and \mathcal{A} comprise contributions of the different sectors with squared reflection coefficients, and that, as usual [12], the coefficients in vacuum channel of \mathcal{M} are geometric means of them. All this, of course, if the D charges sit at the fixed points, and if a corresponding quantization holds for the Wilson lines acting on N charges, that I have not displayed in eq. (22). In space-time models, tadpole conditions related to untwisted massless exchanges constrain the dimensionality of the charge space, whereas others related to twisted exchanges constrain the breaking pattern. Actually, the choice in eqs. (21) and (22) is not unique, and an additional one may be obtained interchanging $Tr(\mathbf{1})$ and $-Tr(\mathbf{R})$, while identifying pairs of Chan-Paton charge spaces. This is one more instance of the phenomenon discussed at length in [12] and [27] and already encountered in eq. (8), whereby pairs of “real” charge spaces associated to orthogonal or symplectic groups are traded for pairs of conjugate spaces of “complex” charges associated to unitary groups.

In the open sector T duality is not necessarily a symmetry, since it involves the interchange of N and D charges, but it can be if identical breaking patterns are chosen for the N and D charge spaces. In the former, the breaking is controlled by (quantized) Wilson lines, while in the latter it is determined by the distribution of D charges among the two fixed points. This has been discussed at length in [21–23]. In modern parlance, this model exhibits the coexistence, again in the

perturbative spectrum, of D branes of different dimensions. Part of this structure was anticipated long ago in the low-energy analysis of [41], that motivated the string analysis of [11].

3. TENSOR MULTIPLETS IN SIX DIMENSIONS

Let me now turn to six dimensions, and in particular to type-I models with simple supersymmetry. They are descendants of the *unique* $K3$ reduction of the Type-IIB string [42], whose low-energy dynamics is described by $N = 2b$ supergravity [43] with 21 $N = 2b$ tensor multiplets⁴ and a moduli space $SO(5, 21)/SO(5) \times SO(21)$. There are only two relevant $N = 2b$ multiplets: the gravitational multiplet includes two left-handed Weyl gravitini, the gravitational field and five self-dual two-tensors, while the tensor multiplet includes a single antiself-dual tensor, two right-handed Weyl spinors and five scalar fields. The $N = 2b$ supergravity was constructed, to lowest order in Fermi fields, in [43], while the unique anomaly-free spectrum was identified in [44].

In proceeding to the open descendants, the novelty is that the projection can result in models with simple supersymmetry in six dimensions with different numbers of residual two-tensors. In simple supersymmetry, the gravitational multiplet includes the gravity field, a left-handed Weyl gravitino and a self-dual two-form, while the tensor multiplet includes an antiself-dual two-form, a right-handed Weyl spinor and a scalar. In this article we shall also consider the vector multiplet, that includes a vector and a left-handed Weyl spinor.

There is a particularly simple class of models, originally constructed in [12], where I first came across the generalized Green-Schwarz mechanism. Many similar models have been recently discussed in [22–24]. In particular, [24] discusses the only string vacuum I am aware of with *no* tensor multiplets at all. Our simple example is built starting from the toroidal reduction of the Type-IIB

⁴In this article I am abiding to present-day notation, whereas in [43] and [13] simple supersymmetry was referred to as $N = 2b$.

string on the $SO(8)$ lattice. In the notation of the previous Section, the torus amplitude reads

$$\mathcal{T} = |V_8 - S_8|^2 \left(|O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2 \right). \quad (23)$$

The next step is a conventional Z_2 orbifold [9], where one reverses simultaneously the sign of the four internal fermions ψ^i and of four of the eight internal fermions that bosonize the internal coordinates. Precisely in the spirit of the fermionic construction in [1], this is manifestly compatible with the corresponding fermionic supercurrent. The characters break accordingly into pairs with opposite eigenvalues under the Z_2 projection. Thus, for instance,

$$V_8 - S_8 \rightarrow (V_4 O_4 - C_4 C_4) + (O_4 V_4 - S_4 S_4), \quad (24)$$

where the two terms, denoted Q_o and Q_v in [12] have, respectively, positive and negative eigenvalues under the involution, and the resulting torus amplitude for the Type-IIB string on the orbifold,

$$\mathcal{T} = \sum_{i=1}^{16} |\chi_i|^2, \quad (25)$$

is a diagonal combination of 16 characters. The identity is

$$\chi_1 = Q_o O_4 O_4 + Q_v V_4 V_4, \quad (26)$$

and the remaining characters are simply determined after a modular S transformation of this expression. Building the “standard” descendant is also simple, since the “parent” model is diagonal. Indeed, in this case Cardy [45] has related the spectrum of boundary operators to the fusion-rule coefficients, an observation turned in [12] into a set of rules for building descendants of rational models. Thus, the open sector corresponding to (25) includes 16 charge sectors and

$$\mathcal{A} = \frac{1}{2} \sum_{ijk} n^i n^j N_{ij}^k \chi_k, \quad (27)$$

where N_{ij}^k are the fusion-rule coefficients. Taking into account the five tadpole conditions, one is left with a number of models, the simplest of which has gauge group $Sp(8)^{\otimes 4}$.

The closed spectrum of this class of models includes five tensor multiplets, and a direct analysis

of the anomaly polynomial led to a big surprise. Its irreducible part cancels as usual after imposing the tadpole conditions, but one is left with the residual contribution

$$I_A \sim \sum_{x,y} \eta_{rs} c_x^r c_y^s Tr_x F^2 Tr_y F^2, \quad (28)$$

where the labels refer to the simple factors of the gauge group, that *does not* factorize. Here η is a Minkowski metric of signature $(1 - n_T)$, with n_T the number of tensor multiplets, and the c 's are constants. I have not indicated explicitly the gravitational contribution, but this could be done extending the range of (x, y) , at the price of including corresponding higher-derivative terms. This setting is thus more general than the original one considered by Green and Schwarz [7], since it demands Chern-Simons couplings for *several* antisymmetric tensors. Moreover, since the anomaly polynomial is quadratic, for the gauge part the mechanism is realized at the level of two-derivative terms. The analysis of the corresponding low-energy supergravity is thus bound to present some peculiarities, and indeed it does. In the first of [13] I constructed its field equations to lowest order in the Fermi fields requiring that the resulting effective action be invariant under supersymmetry. On the other hand, in the second paper of [13] the field equations are derived from the Wess-Zumino consistency conditions [46], extending previous studies of anomalous models in global supersymmetry [47]. The choice affects the gauge current, whose divergence gives in the former case the “covariant” anomaly and in the latter the more satisfactory “consistent” anomaly. The current is *not* conserved, since in this model fermion loops *are to* cancel the residual, factorized anomaly. Of course, this is just Noether's theorem at work, and similar considerations apply to the Green-Schwarz setting of [7], where this inconsistency is present in higher-derivative couplings.

For each choice for the gauge current J , the vector field equation takes the form

$$v_r c_z^r D_\mu F_z^{\mu\nu} = J_z^\nu \quad (29)$$

for each factor z of the gauge group, where v_r are (redundant) coordinates for the scalar fields of the

tensor multiplets, that live on the hyperboloid

$$\eta^{rs} v_r v_s = 1 \quad . \quad (30)$$

Since this describes the coset $SO(1, n_T)/SO(n_T)$, the $SO(1, n_T)$ symmetry is spontaneously broken in the vacuum to $SO(n_T)$. Even the latter symmetry, however, is affected by the couplings c_z^r induced by the anomaly, and as result the effective gauge couplings

$$\frac{1}{g_{(z)}^2} = v_r c_z^r \quad (31)$$

in the vector kinetic term corresponding to (29),

$$L = -\frac{e}{2} v_r c_z^r \text{tr}_z (F_{\mu\nu} F^{\mu\nu}) \quad , \quad (32)$$

can blow up at some points of moduli space [13]. This unusual feature, exploited in [14] in the context of six-dimensional heterotic-heterotic dualities, was ascribed in [15] to a novel phenomenon: the singularities signal the presence, in the vacuum, of a new type of excitation, a tensionless string. The argument is based on the structure of six-dimensional central charges and on the second-order tensor equation of [13], whose source term is the instanton density. In this analogue of the familiar Nambu-Jona Lasinio-Goldstone transition, where a particle becomes massless, one expects a whole tower of higher-spin excitations to play a role. This is the second of the peculiarities alluded to in the Introduction.

Let me stress once more that the vector couplings to the scalars of tensor multiplets induced by the anomaly are revealed by a *perturbative* analysis of type-I vacua, where the dilaton sits in a hypermultiplet. On the other hand, they are quite surprising if referred to the heterotic string, where the dilaton sits in a tensor multiplet, since the vector kinetic terms corresponding to eq. (29) contain non-perturbative contributions [14,15]. For instance, if a single tensor multiplet is present one can solve the constraint of eq. (30) in terms of hyperbolic functions of a scalar. In the heterotic case, this is to be identified with the dilaton, and the vector kinetic term corresponding to eq. (29)

$$L_V = -\frac{e}{2} (c_0 e^{-\phi} + c_1 e^{\phi}) \text{tr} F_{\mu\nu} F^{\mu\nu} \quad (33)$$

includes the non-perturbative coupling described by c_1 .

A simple formula, first derived in [26], follows from the ten-dimensional duality conjecture of [18] and relates the dilatons of heterotic and Type-I vacua:

$$\phi_I^{(d)} = \frac{6-d}{4} \phi_H^{(d)} - \frac{d-2}{16} \log \det G_H^{(10-d)} \quad . \quad (34)$$

It should be noted that the two dilatons are independent precisely in six dimensions. Though quite surprising from the heterotic viewpoint, it is thus natural to construct Type-I vacua *without* tensor multiplets [24]. More amusingly, one is naturally led to speculate that, in their duals, the “missing” dilaton has been effectively stabilized by non-perturbative effects. This point is worthy of a closer look, since it would realize in a simple setting a phenomenon long advocated for four-dimensional string physics.

4. THE SYMMETRY OF BOUNDARY STATES

I shall conclude with a brief discussion of the last exotic phenomenon mentioned in the Introduction, while confining my attention to the rational descendants of a rational conformal theory. In principle, one would expect that the open sector, determined by reflections at the ends of the tube, inherit at most the diagonal part of the symmetry algebra (Virasoro or extended) $A \times \bar{A}$ of the bulk theory. Rather surprisingly, however, in some instances the symmetry of boundary states turns out to be larger. This phenomenon was first noted [28] in the open descendants of the D_5 model of the *ADE* classification of [29].

It is convenient to assume that the bulk spectrum has been resolved into independent components, so that the torus partition function is a permutation invariant built out of m characters,

$$\mathcal{T} = \sum_{i=1}^m \bar{\chi}_i \chi_{\sigma(i)} \quad . \quad (35)$$

The relevant techniques have been developed in [48]. In the diagonal case⁵, the canonical choice

⁵Diagonal means, effectively, that \mathcal{T} is built from the charge-conjugation modular invariant, *i.e.* $\sigma(j) = c(j)$.

for the annulus of eq. (27) [45,12] reflects the one-to-one correspondence between chiral sectors of the bulk spectrum and boundary states. It is also nicely consistent with factorization [12], since the Verlinde formula guarantees that in the corresponding vacuum channel $\tilde{\mathcal{A}}$, exposed by an S modular transformation, each sector enters with a coefficient that is a perfect square:

$$\tilde{\mathcal{A}} \sim \sum_i \chi_i \left(\sum_j \frac{S_{ij} n^j}{\sqrt{S_{1i}}} \right)^2. \quad (36)$$

In Conformal Field Theory, the annulus amplitude may be regarded as a generating function for the multiplicities of boundary operators that mediate between boundary states. Thus, in the diagonal case each chiral sector has m independent reflection coefficients for the m choices of boundary states at the ends of the tube, all expressible in terms of the S matrix.

The A_7 model of [29]

$$\begin{aligned} \mathcal{T}_{A_7} = & |\chi_1|^2 + |\chi_2|^2 + |\chi_3|^2 + |\chi_4|^2 + |\chi_5|^2 \\ & + |\chi_6|^2 + |\chi_7|^2, \end{aligned} \quad (37)$$

where the labels of the seven level-six characters are $2I+1$, with I their isospins, is a convenient example with unextended symmetry. On the other hand, the D_6 model of [29] has level eight and

$$\mathcal{T}_{D_6} = |\chi_1 + \chi_9|^2 + |\chi_3 + \chi_7|^2 + 2|\chi_5|^2, \quad (38)$$

is a simple model with extended symmetry. In this case, there is a fixed-point ambiguity, resolved assigning *two* different fields to the term with multiplicity 2. Together with the other combinations of level-eight $SU(2)$ characters in eq. (38), these define a fusion algebra with some N_{ij}^k larger than one. As a result, some terms occur in \mathcal{A}_{D_6} with multiplicities. This result reflects the existence of inequivalent three-point functions. The simplest instance of this phenomenon is quite familiar: in $SU(3)$ there are two invariant tensors with three adjoint indices, the structure constants f^{abc} and the symmetric tensor d^{abc} .

In open descendants of off-diagonal models, the direct correspondence between boundary states and chiral sectors of the bulk spectrum is lost,

since in this case the tube can only support a portion of them. In particular, *if the boundaries are to preserve the bulk symmetry, the allowed sectors are those paired with their conjugates in the torus modular invariant*. Thus, some of the reflection coefficients vanish, a condition naturally met by suitable linear combinations of the “diagonal” boundary states. However, *not all boundary states are obtained in this way*. For instance, in all D_{odd} models one has a simple current of half-integer dimension, and the proper boundary states are combinations of pairs of “diagonal” ones invariant under the action of this simple current [28]. This is quite reminiscent of the structure of the first two terms in eq. (38), and is effectively an extension by the simple current, that in these models has non-integer dimension! As in the more familiar bulk case, the boundary state naively invariant under the action of the simple current actually splits into a pair of states. For instance, in the D_5 case the torus modular invariant is built out of the level-six characters of eq. (37), but

$$\begin{aligned} \mathcal{T}_{D_5} = & |\chi_1|^2 + |\chi_3|^2 + |\chi_5|^2 + |\chi_7|^2 \\ & + |\chi_4|^2 + \chi_2 \bar{\chi}_6 + \chi_6 \bar{\chi}_2, \end{aligned} \quad (39)$$

and therefore one would expect a total of five boundary states. However, there are only three combinations of “diagonal” boundary states invariant under the action of the simple current, $(1+7)$, $(2+6)$ and $(3+5)$. The correct total number, five, is thus recovered precisely because the state corresponding to the middle field splits into a pair 4_+ and 4_- . The construction in [27] recovers quite naturally the pair of split fields, but there is also an analogue of the multiple three-point functions of conventional bulk extensions. This is directly induced by the fusion of pairs of “off-diagonal” boundary states, and has a direct consequence for the annulus amplitude, that contains multiplicities. For instance, the conventional (real) choice for the D_5 annulus amplitude

$$\begin{aligned} 2\mathcal{A}_{D_5} = & \chi_1(n_1^2 + n_2^2 + n_3^2 + n_{4_+}^2 + n_{4_-}^2) + \\ & (\chi_2 + \chi_6)(2(n_1 n_2 + n_2 n_3 + n_{4_+} n_3 + n_{4_-} n_3)) + \\ & \chi_3(n_2^2 + 2n_3^2 + 2(n_2 n_{4_+} + n_2 n_{4_-} + \\ & n_{4_+} n_{4_-} + n_1 n_3)) + \chi_4(4n_2 n_3 + 2(n_1 n_{4_+} + \end{aligned}$$

$$n_{4+}n_3 + n_1n_{4-} + n_{4-}n_3) + \chi_5(n_2^2 + n_{4+}^2 + n_{4-}^2 + 2n_3^2 + 2(n_2n_{4+} + n_2n_{4-} + n_1n_3)) + \chi_7(n_1^2 + n_2^2 + n_3^2 + 2n_{4+}n_{4-}) \quad , \quad (40)$$

contains multiplicities in χ_3 and χ_5 , and the whole D_{odd} series follows a similar pattern.

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